INDIAN STATISTICAL INSTITUTE

CHENNAI CENTRE

M.STAT FIRST YEAR 2016-17 SEMESTER-I

STATISTICAL INFERENCE

Mid Semester Examination

Total Marks : 60 Duration : 3hours

Answer any Six Questions

(1) Let $(X_1, \dots, X_n)$ be a random sample from a distribution on $\mathcal{R}$ having the Lebesgue density $\exp^{-(x-\theta)} I_{(\theta,\infty)(x)}$, where $\theta > 0$ is an unknown parameter.
   (a) Show that the minimal sufficient statistic is complete. Use Basu's Theorem to show that $X_{(1)}$ and S^2 are independent.

(2) Let X be a discrete random variable with

$$P_\theta(X = x) = \frac{\binom{\theta}{x}\binom{N-\theta}{n-x}}{\binom{N}{n}} \quad x = 0, 1, 2, \dots, \min\{\theta, n\},\ n-x \le N-\theta$$

where n and N are positive integers, $N \ge n$, and $\theta = 0, 1, \dots N$. Show that X is complete.

(3) Let $(X_1, \dots, X_n)$ be a random sample from $N(\mu, \sigma^2)$, σ^2 - known. Find the best unbiased estimate of μ^3 and μ^4.

(4) Let $(X_1, \dots, X_n)$ be a random sample from

$$f_\theta(x) = \Gamma(\theta)x^{\theta-1} \quad 0 \le x \le 1,$$

where $0 < \theta < \infty$. Find the MLE of θ. Show that the variance of MLE goes to zero as n goes to ∞.

(5) Let $(X_1, \dots, X_n)$ be a random sample from bernoulli $\{p\}$.
   (a) Show that the variance of MLE attains the Cramer- Rao lower bound.
   (b) For n=4, Show that the product $X_1X_2X_3X_4$ is an unbiased extimate of P^4 and use this fact to produce UMVUE of P^4.

(6) Let $(X_1, \dots, X_n)$ be a random sample from Poisson (λ) and λ having a gamma (α, β) prior distribution .
   (a) Find the posterior distribution of λ.
   (b) Find the posterior mean and variance.

(7) Let $f(x|\theta)$ be the logistic distribution $\exp^{-(x-\theta)}/[1+\exp^{-(x-\theta)}]^2$,where $\theta > 0$ is an unknown parameter.
   (a) Show that the family has an MLR property.
   (b) Based on one observation , find the most powerful test of $H_0 : \theta = 0$ versus $H_1 : \theta = 1$. For $\alpha = 0.2$, find the size of the type II error.